A game-theoretic approach towards climate change control

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Future of Energy Seminar

Non-cooperative game theory I: Prisoner's dilemma

Dresher & Flood (1950), Tucker (1950)

	II: do not confess	II: confess
I: do not confess	Both 2 years	I: life; II: free
I: confess	I: free; II: life	Both 20 years

Cooperative outcome unreachable under rational decision making

Non-cooperative game theory II: Nick Vriend game

	Trafalgar Square	Oxford Circus	Hyde Park Corner	Covent Garden	. . .
Trafalgar Square	1, 1	0, 0	0, 0	0, 0	0, 0
Oxford Circus	0, 0	1, 1	0, 0	0, 0	0, 0
Hyde Park Corner	0, 0	0, 0	1, 1	0, 0	0, 0
Covent Garden	0, 0	0, 0	0, 0	1, 1	0, 0
...	0, 0	0, 0	0, 0	0, 0	1, 1

Nash equilibria

Nash equilibrium agreements are self-enforcing

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Unique Nash equilibrium

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Multiple Nash equilibria

- Political freedom, limited, but not determined, by economic considerations
- Negotiations are necessary

Results

Analysis of transboundary pollution game

Determination of the set of *all* Nash equilibria [1]

Theoretical upper bound efficiency International Environmental Agreements

Literature so far has focused on the *least efficient* equilibrium

[1] caveats apply

Programme

- Transboundary pollution game
- Unique continuous equilibrium
- All non-continuous equilibria
- Efficiency gains

The transboundary pollution game

van der Ploeg & de Zeeuw (1992)

Emissions $q_i(t)$ of N countries affect stock $x(t)$ of pollutant

$$\dot{x}(t) = \sum_{i=1}^{N} q_i(t) - \delta x(t), \quad x(0) = x_0$$

Value for country i

$$V_i(x_0) = \max_{q_i} \int_0^\infty e^{-\rho t} \left(\underbrace{\alpha q_i(t) - \frac{\beta}{2} q_i(t)^2}_{\text{benefits industrial production}} \quad \underbrace{-\frac{\gamma}{2} x(t)^2}_{\text{pollution costs}} \right) \mathrm{d}t$$

Calibration: assumptions

- Business-as-usual emissions: 10 GtC/y.
- Current CO_2 pollution: 0.5 TtC
- Carbon budget: 1.0 TtC
- Current Social Cost of Carbon: 400 $/tC
- Discount rate: $\rho = 0.025$ y^{-1}
- Natural decay: $\delta = 0.001$ y^{-1}

Calibration: results

Coefficients

$$\alpha = 678\ \text{T\$/TtC}, \quad \beta = 339 \cdot 10^3\ \text{T\$ y}/\text{TtC}^2, \quad \gamma = 1.953\ \text{T\$}/\text{y TtC}^2$$

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Natural units

$$[\text{value}] = \frac{\alpha^2}{\sqrt{\beta\gamma}} = 565\,\text{T\$}, \qquad \text{2022 Global GDP} : 101\,\text{T\$}$$

$$[\text{time}] = \sqrt{\frac{\beta}{\gamma}} = 417\,\text{y},$$

$$[\text{pollutant}] = \frac{\alpha}{\sqrt{\beta\gamma}} = 0.833\,\text{TtC}$$

After rescaling to natural units: $\alpha = \beta = \gamma = 1$, $\rho = 6.25$, $\delta = 0.42$.

How is the game played?

Markov assumption

The state variable $x(t)$ contains all relevant information about the system

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Autonomous Markov strategies

Emission strategies only condition on the state

$$q_i(t) = \phi_i(x(t))$$

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Best reply

Country i chooses best strategy given the actions of all others

$$\phi_i = \mathscr{B}_i(\phi_1, \ldots, \phi_{i-1}, \phi_{i+1}, \ldots, \phi_N) = \mathscr{B}_i(\phi_{-i})$$

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If this holds for all i, then $(\phi_1, \ldots, \phi_N)$ is a Nash equilibrium

Finding Nash equilibrium strategies

Finding Nash equilibrium strategies

Hamilton–Jacobi equation:

$$\rho V_i(x) = \max_{q_i}\left[q_i - q_i^2/2 - x^2/2 + V_i'(x)\left(q_i + \sum_{j\neq i}\phi_j(x) - \delta x\right)\right] \;\Rightarrow\; q_i = 1 + V_i'(x)$$

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Symmetry assumption: $V_i = V$ and $\phi_i = 1 + V'$ for all i

$$\rho V(x) = -\tfrac{1}{2}x^2 + (1 + V'(x))^2/2 + V'(x)\big[(N-1)(1+V'(x)) - \delta x\big]$$

Finding Nash equilibrium strategies

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Differentiation

$$\rho V'(x) = -x + V'(x)[(N-1)V''(x) - \delta] + V''(x)\big[N(1+V'(x)) - \delta x\big]$$

Finding Nash equilibrium strategies

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Differentiation

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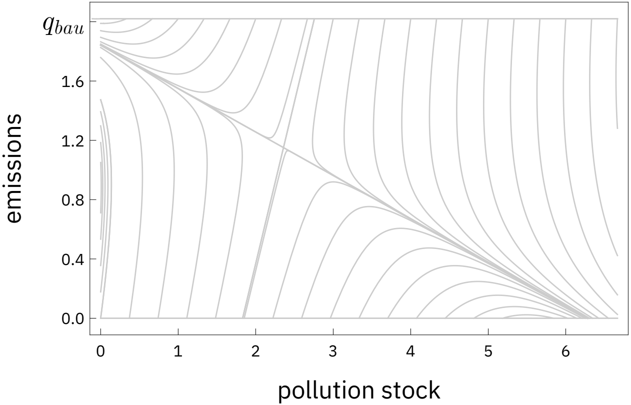
Markov strategy $\phi = 1 + V'$ satisfies

$$\phi'(x) = \frac{(\rho+\delta)(\phi(x)-1) + x}{(2N-1)\phi(x) - (N-1) - \delta x}$$

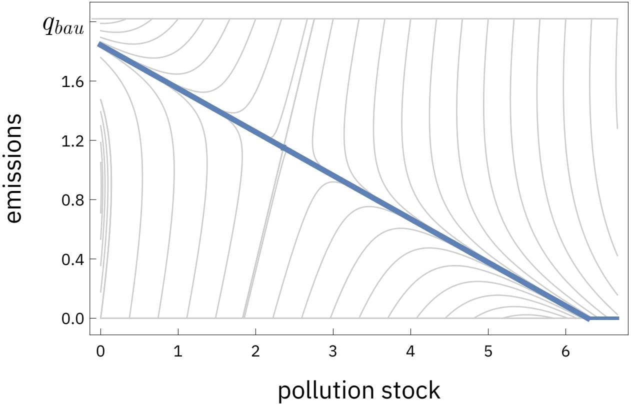
Nash equilibria



Tsustui & Mino (1990)
Dockner & Ngo Van Long (1993)

- Many candidates for equilibrium emission strategies

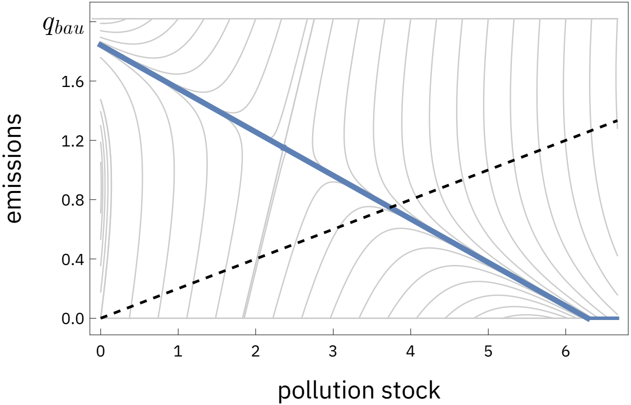
Nash equilibria



Starr & Ho (1969)
van der Ploeg & de Zeeuw (1992)

- Unique continuous Nash equilibrium $q = \phi_{\text{lin}}(x)$
- Globally defined
- (Piecewise) linear

Nash equilibria



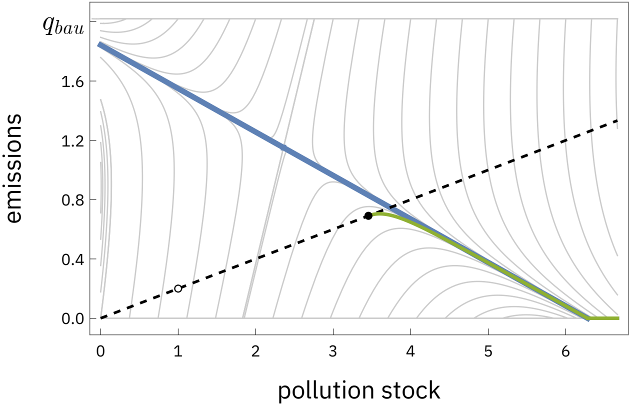
Dynamics:

$$\dot{x} = N\phi(x) - \delta x$$

Steady states on line

$$Nq - \delta x = 0$$

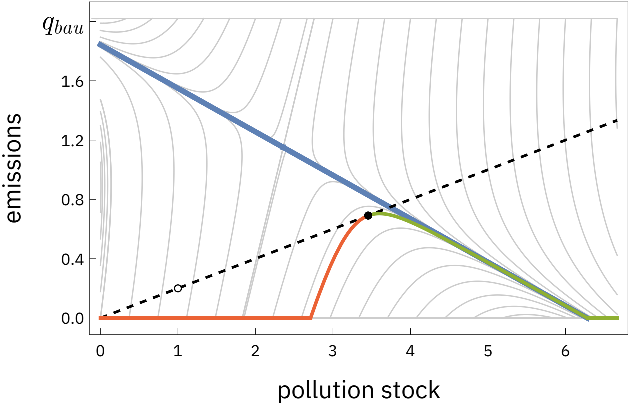
Nash equilibria



Dockner & Ngo Van Long (1993)

- Multiple locally defined Nash equilibria
- Most efficient steady state tends to cooperative steady state as discount rate $\rho \to 0$

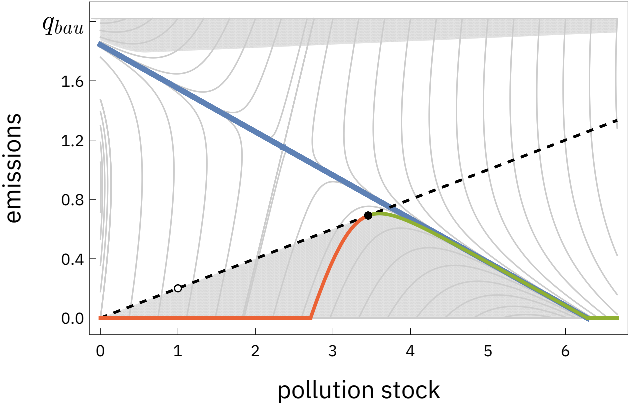
Nash equilibria



Rowat (2007)

- Nash equilibrium strategies have to be defined globally

Nash equilibria

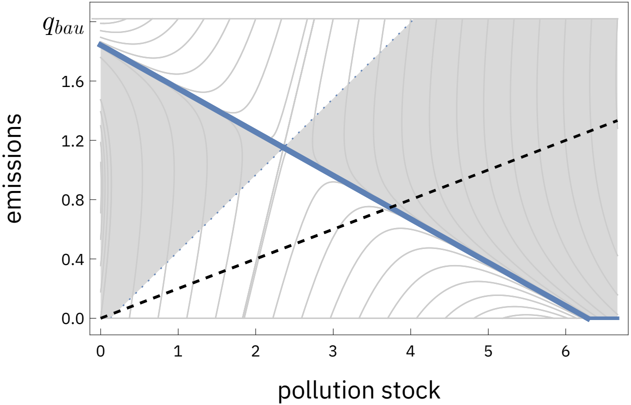


Implications of general theory I

'Unsustainable limit' region

- Jumps cannot occur
- Unsustainable limit behaviour

Nash equilibria

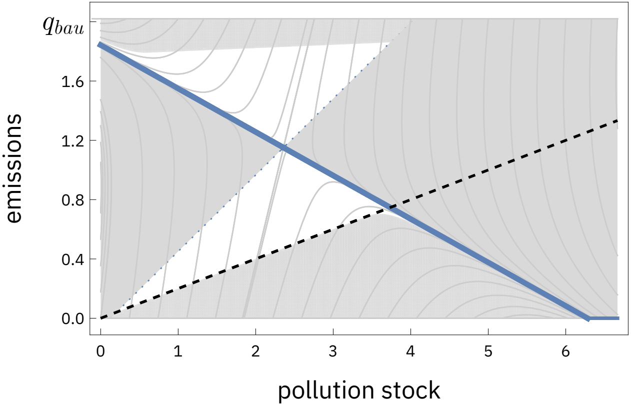


Implications of general theory II

'No continuation' region

- Jumps can occur, but
- Solutions cannot be continued globally

Nash equilibria

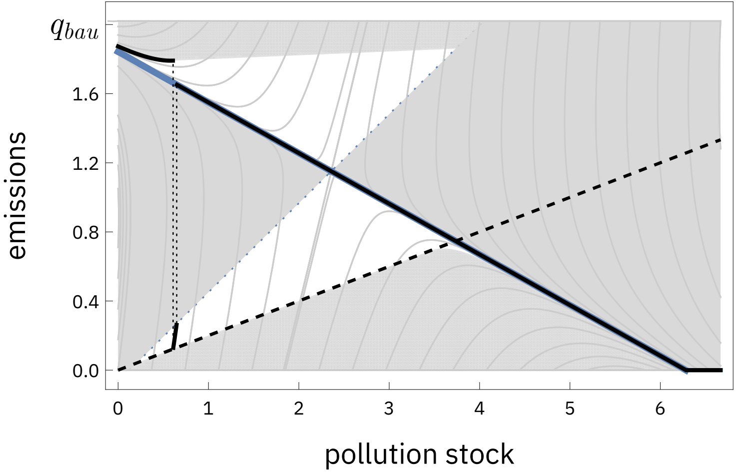


Implications of general theory III

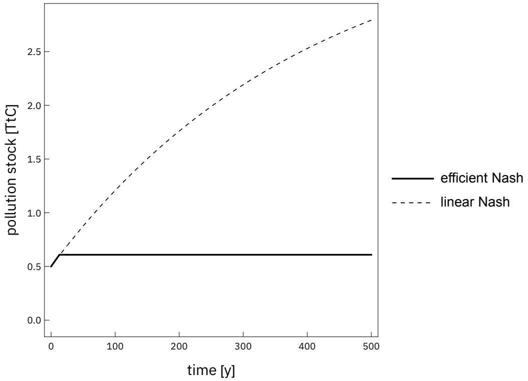
- Through any point of remaining region there pass infinitely many Nash equilibria
- Linear equilibrium has always lowest payoff

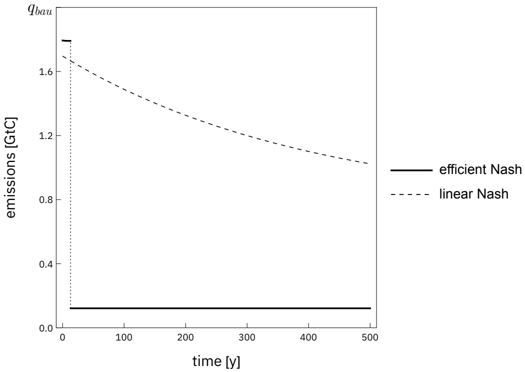
Efficient Nash equilibrium

Pareto-optimal Nash equilibrium for $x(0) = 0.5$ TtC

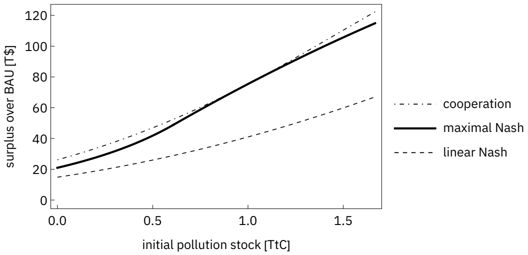


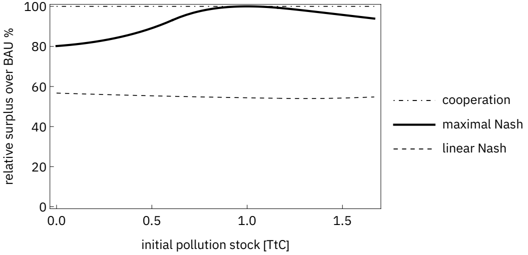
Efficient Nash equilibrium: Stocks and emissions





Maximally efficient Nash equilibria





Self-enforcing International Environmental Agreements ($=$ Nash equilibria)

Efficiency: 80%–100% of full cooperation

Conclusion

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Prisoner's dilemma: competition vs cooperation

- Unique Nash equilibrium
- Selfish competition prevents reaching optimal cooperative outcomes

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Prisoner's dilemma: competition vs cooperation

- Unique Nash equilibrium
- Selfish competition prevents reaching optimal cooperative outcomes

Alternative: coordination

- Many Nash equilibria, some with excellent outcomes
- Good equilibria may require drastic policies
- Communication and negotiations are necessary to coordinate